UK Junior Mathematical Olympiad 2006

Organised by The United Kingdom Mathematics Trust

Tuesday 13th June 2006

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

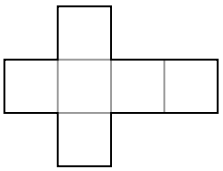
   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

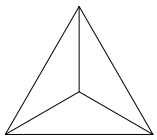
A1 What is the value of $1 + 2 \times (3 + 4^5) + 6 + 7 - 8 \times 9 + 10$?

A2 The perimeter of this net of a cube is 42cm.
What is the volume of the cube?



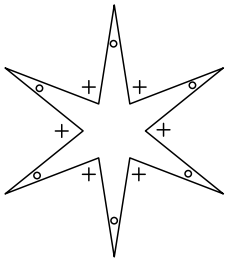
A3 Sarah writes down all the three-digit positive integers for which the product of their digits equals 36. What is the difference between the greatest and the smallest of these numbers?

A4 An equilateral triangle is drawn on a sheet of white card and divided into three identical regions as shown. Then each region is painted red or yellow or blue. More than one region may be painted in the same colour.
How many different painted triangles can be made in this way?
(Rotating a triangle does not make it different.)



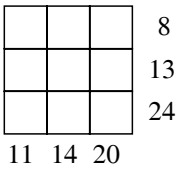
A5 A balloon seller starts the day with a certain number of balloons. He then sells one third of his balloons to boys, 20% to girls, and three times the difference between these amounts to adults. At the end of the day, he has eight balloons left. How many balloons did the seller have at the start?

A6 In the diagram the star is made up of equal line segments.
Each of the angles marked + is 70°.
What is the size of the angles marked ∘ ?



A7 Each year on Tom's birthday, his grandfather gives him some pocket money. The amount, in pence, is calculated by multiplying together Tom's age and his grandfather's age on that day.
This year Tom received £7.81. How much did he receive last year?

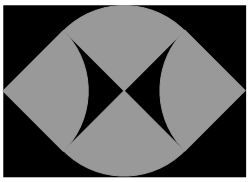
A8 The numbers 1 to 9 are to be placed so that there is one number in each square and the row and column totals are as shown



What number goes in the central square?

A9 The prime number 11 may be written as the sum of three prime numbers in two different ways: 2 + 2 + 7 and 3 + 3 + 5. What is the smallest prime number which can be written in two different ways as the sum of three prime numbers **which are all different**?

A10 In the flag shown alongside, the regions shaded grey are quarter circles.
If the height of the flag is 1m, what is its breadth?



Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

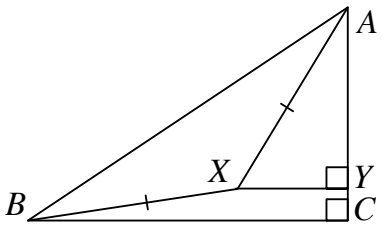
B1 In her purse, Jenny has 20 coins, with a total value of £5. There are three denominations of coin – 10p, 20p and 50p – in her purse and she has more 50p coins than 10p coins. How many of each type of coin does she have?

B2 $$97 \rightarrow 63 \rightarrow 18 \rightarrow 8.$$

An example of a particular type of number chain is shown above. The first number must be a positive integer. Each number after the first is the product of the digits of the previous number, so in this case $63 = 9 \times 7$; $18 = 6 \times 3$; $8 = 1 \times 8$. The chain stops when a single-digit number is reached.

Suppose that in such a chain the final number is 6. Find all possible two-digit first numbers for this chain.

B3 In this diagram, Y lies on the line AC, triangles ABC and AXY are right angled and in triangle ABX, $AX = BX$. The line segment AX bisects angle BAC and angle AXY is seven times the size of angle XBC. What is the size of angle ABC?



B4 Start with an equilateral triangle ABC of side 2 units, and construct three outward-pointing squares $ABPQ$, $BCTU$, $CARS$ on the three sides AB, BC, CA. What is the area of the hexagon $PQRSTU$?

B5 An intelligent bug starts at the point (4, 0) and follows these instructions:

(i) first face "East" and walk one unit to the point (5,0);

(ii) from then on, whenever you arrive at a point (x, y) with x and y both integers,
either turn left through 90° if $x - y$ is a multiple of 4 or is 1 more than a multiple of 4;
or turn right through 90° if $x - y$ is 2 more than a multiple of 4 or is 3 more than a multiple of 4;
and then walk one unit to the next point whose coordinates are both integers.

After one move, the bug is at the point (5,0).

(a) Where will the bug be after 12 moves?

(b) Where will the bug be after 50 moves?

B6 The numbers 1 to 7 are to be placed in the seven regions formed by three overlapping circles, with 6 in the central region, so that there is one number inside each region and the total of the numbers inside each circle is T.

What values of T are possible?